RCA

Application is being made for second class mail rates.

Mathematics News Letter

SUBSCRIPTION $1.00 PER YEAR Single Copies, 15c

Published under auspices of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

This journal is dedicated to mathematics in general, to the following causes in particular (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of Mathematical Association of America and National Council of Teachers of Mathematics projects.

VOL. 5 BATON ROUGE, LA., FEBRUARY-MARCH, 1931 NO. 6

THE NATCHITOCHES MEETINGS

The officials of the three organizations whose annual joint programs were executed at Natchitoches, Louisiana, on March 13 and 14 are to be congratulated for the highly admirable manner in which the programs were carried out. The Natchitoches hosts left nothing undone that might contribute to the effectiveness and success of the meetings.

One feature of the correlated programs that seemed to us an outstanding one was the frankness with which the high school representative and the college representative faced and discussed some of the vexing questions of articulation between secondary and college mathematics teaching. In this respect, there seemed to be for the first time in the history of the consolidated organizations, a measurable realization of some of the fundamental purposes which had been aimed at when the consolidation was originally effected. —*S. T. S.*

THE NEWS LETTER AND THE NATCHITOCHES MEET

As Chairman Nichols substantially expressed it, while in nearly all previous Section-Council annual meetings the cause of

the Mathematics News Letter had seemed to be lacking many of the elements of an encouraging promise for its future, the Natchitoches meeting was charaterized by a very pronounced atmosphere of confidence that the journal is entitled to and will ultimately acquire and maintain an influential place among the publications aiming to promote mathematics and its teaching. Put otherwise, it was at Natchitoches that for the first time in the history of the little journal the Louisiana-Mississippi group of high school and college mathematicians voted a backing of the News Letter that was both solid and enthusiastic.

It is but fair in this connection to say that a very large measure of credit for this renaissance of interest in the Letter should go to the national officials of the Mathematical Association of America, especially to Secretary Cairns and Professor Slaught, whose unselfish interest in the success of our Louisiana-Mississippi mathematical projects has never flagged, and whose counsel has been invaluable. —*S. T. S.*

A RESTATEMENT OF SOME OBJECTIVES

We believe that it was three years ago, following our second meeting at Jackson, Mississippi, we published in the News Letter what we called "An Outline of Features Proposed for the Mathematics News Letter." A careful re-reading of this "Outline" has impressed us with the idea that a re-statement of it would not only be proper but would also be useful in view of the fact of a very greatly increased responsibility for keeping the content of the journal oriented definitely in respect to certain fixed fundamental objectives.

It is in accord with this idea, then, that we here offer the following reprint of those objectives of the Letter which pertain more to the teaching and administrative phases of mathematics than to strictly technical mathematics.

1. Articles dealing with what may broadly be described as the correlation of high school and college mathematics courses. Under this head systematic effort will be made to publish in clear and popular style constructive discussion of

(a) Improvement in schemes of articulation between secondary and college mathematical courses and programs.

(b) Comparative studies of these schemes as they now exist in different states of the Union.

(c) How high school mathematics programs may furnish a maximum service to colleges.

(d) How college programs in mathematics may best serve the high schools.

(e) Comparative study of college and high school and college objectives in mathematics teaching.

2. Sectioning classes in mathematics according to the different grades of ability.

3. Mathematics in its relation to the so-called social sciences.

4. The principle of election and mathematics in a liberal arts college program.

5. The mathematical mind and the non-mathematical mind.

6. Methods of motivating interest in mathematics.

7. History of mathematics and its applications.

8. Sketches of successful mathematicians.

9. Mathematics and school administrations.

10. Mathematics as culture.

11. Mathematics as discipline.

12. Mathematics as a tool.

13. Personality in mathematics teaching.

14. Problem solving department.

15. Review of important articles in the Mathematics Teacher.

16. Review of important articles in the American Mathematical Monthly.

17. M. A. of A. programs and projects.

18. Programs and projects of the National Council of the Teachers of Mathematics.

19. Promoting the annual meeting of Louisiana-Mississippi Section and Council.

20. News notes from college and high school Mathematics departments of Louisiana and Mississippi.

21. Increasing the News Letter Subscriptions.

22. The Work of the Committee on Mathematics Requirements.

23. Correlation between mathematics grades and grades in other subjects. —*S. T. S.*

SOME LIBRARY SUBSCRIBERS

Friends of the Letter will be interested and possibly surprised to see how wide a range is covered by the following list of our college and high school library subscriptions:

Allegheny college, Meadville, Pa..
Antioch College, Yellow Springs, Ohio.
Beloit College, Beloit, Wis.
College of Pacific, Stockton, Cal.
Delta State Teachers College, Cleveland, Miss.
Defiance College, Defiance, Ohio.
East Central State Teachers College, Ada, Okla.
New Orleans Normal School, New Orleans, La.
Isidore Manual Training School, New Orleans, La.
Loyola University Students' Library, New Orleans, La.
Louisiana Normal College, Natchitoches, La.
LeCompte High School, LeCompte, La.
Merdian High School, Merdian, Miss.
Louisiana State University, Baton Rouge, La.
Manchester College, Manchester, Ind.
New Mexico State Teachers College, Silver City, N. M.
Purdue University, Lafayette, Ind.
Peru State Teachers College, Peru, Ind.
Queens College, Charlotte, N. C.
Rice Institute, Houston, Tex.
Ruston High School, Ruston, La.
Steven F. Austin Teachers College, Nacogdoches, Tex.
University of Washington, Seattle, Wash.
State Teachers College, Upper Montclair, N. J.
St. Mary's Dominican College, New Orleans, La.
State Teachers College, Milwaukee, Wis.
Massachusetts Institute of Technology, Cambridge, Mass.
Centenary College, Shreveport, La.
Louisiana Polytechnic Institute, Ruston, La.
A. M. College, Starkeville, Miss.

University of Mississippi, University, Miss.
Tulane University, New Orleans, La.
Gulfport High School, Gulfport, Miss.
May the list grow! —*S. T. S.*

GENUINE PROFESSIONAL SPIRIT!

We take the liberty of publishing the two following letters without having first requested the writers of the letters to permit us to do so. We do not believe that either of them will object to our thus sending them on a service mission:

Hattiesburg, Miss., April 4, 1931.

Professor S. T. Sanders,
Louisiana State University,
Baton Rouge, La.

Dear Sir:

Please enter my subscription to the Mathematics News Letter. If the issues from September, 1930, are available, I should like to have my subscription dated then so that I may receive all numbers of the present session.

I am encouraging all our graduates in mathematics to subscribe, for I appreciate the unique project which La.-Miss. Mathematics teachers are undertaking.

Very truly yours,
MRS. MARTIN L. RILEY,
Critic Teacher of Mathematics.

Mason, Texas, Oct. 28, 1930.

"Mathematics News Letter"
Baton Rouge, La.

Gentlemen:

In the "Texas Mathematics Teachers' Bulletin" I read a commendation of your publication. As a teacher of mathematics, I am interested in articles which deal with the problems of High Mathematics. Please give me information concerning the nature of the publication, the price of the subscription, etc.

Sincerely,
(Miss) MARGARET FISK.

* HIGH SCHOOL AND COLLEGE MATHEMATICS

By R. L. O'QUINN,
Louisiana State University

To anyone who considers the problems which arise in the proper handling of high school and college mathematics, it must occur that the aims and purposes for which the various branches of mathematics are taught are of primary importance, and that one must have clearly in mind at all times the goal of his efforts if his work is to be crowned with any measure of success. Since the high school teacher expects and urges the best of his product to continue their work in the colleges and universities, and since it is of supreme importance to the college teachers that the freshmen shall have had a type of training which will enable them to do successfully the work required in college, it becomes necessary for the college and high school teachers to consider together the ends to be accomplished, and the means whereby those ends are to be brought about. In order to do this, we should calmly and dispassionately consider every element of the problem, and together work out its solution. The ends to be achieved in the teaching of mathematics do not exist for the high schools alone, nor alone for the higher educational institutions. They are the ends to be achieved in the study of mathematics in general, and the main ends toward which we are striving begin in the grades and continue throughout our years of study; that is, the more general aims of mathematical study.

Let us examine some specific aims of the study of high school mathematics. I assume that the aims set forth in the Report by the National Committee on Mathematics Requirements, under the auspices of the Mathematical Association of America express the most advanced thought on the subject. I shall quote this report at some length. They were divided into three classes: (1) Practical or utilitarian, (2) disciplinary, (3) cultural. The three classes of aims were not regarded, however, as mutually exclusive.

The utilitarian aims, which are to be achieved in the secondary schools, I shall not enumerate here. Among the cultural aims are the following:

*Read at the 1931 Annual Meeting of the Louisiana-Mississippi Chapter of N. C. of T. of M. held at Natchitoches, La.

1. Appreciation of beauty in the geometrical forms of nature, art, and industry.

2. Ideals of perfection as to logical structure, precision of statement and of thought, logical reasoning, discrimination between true and false, etc.

The following are mentioned among disciplinary aims:

1. The acquisition, in precise form, of those ideas or concepts in terms of which the quantitative thinking of the world is done.

2. The development of ability to think clearly in terms of such ideas and concepts.

(a) Analysis of a complex situation into its simpler parts.

(b) The recognition of logical relations between inter-dependent factors and the understanding and expression of such relations in precise form.

(c) Generalization; that is, discovery and formulation of a general law, together with its properties and application.

3. The acquisition of mental habits and attitudes which will make the above training effective in the life of the individual.

To quote the Report: "The practical aims enumerated, in spite of their vital importance, may without danger be given a secondary position in seeking to formulate the general point of view which should govern the teacher, provided only that they receive due recognition in the selection of material and that the minimum of technical drill is insisted upon.

The primary purpose of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and space which are necessary to an insight into and control of our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual."

No doubt the most serious obstacle in the carrying out of the aims set forth for the high schools is the lack of a sufficiency of teachers of mathematics who have the academic and professional training, the personality, and the enthusiasm for the teaching of the subject. A good many states, notably in the West, are requiring at least two years of college work in mathematics before teaching the subject in high school. The next few years will see a great step forward in respect to the preparation requirements.

To quote from the Report: "Moreover, the recognized position of the teacher in the community must be such as to attract men and women of the highest ability into the profession. This means not only higher salaries but smaller classes and more leisure for continued study and professional development. It will doubtless require a time before the public can be made to realize the wisdom of taxing itself sufficiently to bring about the desired result. But if this ideal is continually advanced and supported by sound argument there is every reason to hope that in time the goal may be reached".

It is in the interest of good teaching that the whole profession should stand behind these principles, and work for their realization. Those in executive positions must demand higher standards of scholarship and training, and the public must be convinced that anyone teaching the subject of mathematics should be thoroughly trained for his work. Just anyone cannot teach mathematics. Indeed, there are perhaps as few real teachers of this subject as of any in the curriculum.

What are the aims in teaching college mathematics and how are they related to those of the high school? Specifically, we supply the necessary mathematical training for a proper understanding of the natural sciences, such as physics and chemistry, as well as biology, economics, educational measurements, and to a lesser degree, a good many others in various fields. It is basic in any course in engineering. How can we teach so that our work shall be most effective in widely separated fields, and keep in mind a general aim in our teaching? My contention is that the general aim of college mathematics is merely an extension of the general aim set forth for the teaching of high school mathematics. Even if we look at the subject merely from the standpoint of its utility in the fields already enumerated, we must develop the powers of understanding and analysis of our pupils if our work is to be made effective. It is not so much the amount of subject matter that the student possesses that makes for his success, as it is his power to turn it to account, to understand thoroughly, and be able to apply what he has learned in a new situation. Can it be said that the college teacher always succeeds in achieving this end, while the high school teacher so often fails? I think not.

The freshman entering college misses the individual instruc-

tion to which he has been accustomed, as well as the personal contact of his teachers. He receives instruction in a large group, and too often the personal contact is absent. Consequently, the student may think his teachers have no interest in his success or failure. No matter how good one may be in the mere technique of instruction, it is important to remember that we are dealing with human nature, and that the best results are obtained only when there is a mutual feeling of respect and friendliness on the part of pupil and teacher. It is the pupil that we must teach rather than a book. We have to take the pupils as they come to us, and so teach that the work can be built upon what they already know. The college teacher can blame the high school teacher, the high school teacher, the grade teacher, etc., until finally the poor primary has no alibi except environment of the pupil.

By what methods do we attempt to achieve our aims in the colleges and universities? One method is the lecture method, so called because the teacher commonly occupies the class hour by delivering a lecture to his class. This has been called the easiest and most ineffective method of teaching. Some of the indictments against it are:

1. It hinders initiative and individuality of the pupil.
2. It encourages laziness, since the teacher does too much of the pupil's thinking.
3. The lecturer does not know how much of his lecture the class fails to grasp till after the examination.
4. Lectures are an exposition of fact or argument by the teacher instead of by the pupil, while the pupil tries mainly to absorb what is said.
5. Inevitably, the lecturer comes to discourage questions.

As opposed to the lecture method we may mention the laboratory method, in which the pupils do not work as a group, but each is working independently of the others on his own problems. The teacher passes among the students, giving help where needed. There may or may not be lectures.

As an alternative plan I shall set forth a method which in my experience and observation has been successfully used. I should retain the best features of the lecture method by lecturing when we are dealing with routine work, or work of a simple character such that there is an easy grasp of the subject matter by

the whole group. It can be used effectively at the beginning of a new chapter or subject, when we wish to get immediately before the whole group facts and material with which to reason. This having been done, the teacher stops **telling** and begins **teaching.** If we are to carry out the primary purposes of the teaching of mathematics, if we intend to develop the powers of understanding and analysis of relations of quantity and space we must do it by the most skillful questioning, so that the pupil may be taught to reason, to develop increasing power in getting at the heart of a difficulty, resolving it into its simpler elements and thus analyzing the most complex difficulty. This can be done under the leadership of the teacher at the blackboard, by calling upon his pupils in turn, and so framing his questions as to make the pupil consciously build up a logical train of thought.

Pupils need to be taught to reason, and there is no better way to bring it about than by making a conscious effort at all times to make the logical process paramount in all teaching, rather than the facts that are learned. It goes without saying that a teacher should welcome at all times intelligent questions, and encourage the student to ask them. A pupil may be sent to the board, a proposition may be set up for proof, or a problem for solution. If the pupil can think his way through without help, by all means encourage it, but if help must be given, let the necessary suggestions be drawn from the students themselves by asking good questions rather than by telling. One fact discovered is worth any number given by someone else. Pupils may be sent to the board singly, or in groups to vary the method of the recitation as occasion warrants. If pupil or teacher is doing the writing on the board, by the question method every member of the class can be impelled to bring his whole powers of concentration on the problem at hand.

There has never been found any better means of provoking thought than by continually propounding questions for settlement which are relevant to the subject-matter being studied, whether these questions are proposed by teacher or by student. Needless to say, it is difficult to question skillfully, but no more so than it is for the average person to learn to reason without such a stimulus. It is evident, since the calculus requires more refined processes of thought than does freshman mathematics that if we

do not increase his power to reason in his freshman year, he will suffer for it the next year, etc.

The interest of the pupil is often increased if he is called upon to make a written or oral report to the class on something in which he has the ability and access to sources of information. Among such topics we might mention some of the unique applications of the topic we happen to be studying. Historical information regarding famous mathematicians and their works stimulates the interest of the class.

But what of those specific facts that we expect to make use of in other branches of his course? It is my contention that by laying the main emphasis on the development of the student's powers of analysis and habits of thought, we can make what he learns most effective, and his store of information will be greater and more reliable because it is better organized. It would seem well then for the college teacher, whether or not he has had formal professional training, to give much time and thought to the pedagogical problems which arise in his work.

In conclusion, there are some who say that the ability to reason cannot be taught, that the powers to do logical thinking cannot be increased by conscious, directed effort, and that habits of thought do not carry over from one field of endeavor to another, but this is not borne out by the evidence. There are freshmen entering the university every year of whom I feel confident in advance that they can think and reason logically, even tho their store of information may not be above the average, simply because their teachers are in the habit of laying the stress on developing those thought processes that are effective in the study of this and related fields.

PROBLEM DEPARTMENT

Edited by T. A. BICKERSTAFF,
Universty of Mississippi

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to **T. A. Bickerstaff, University, Mississippi.**

No. 1—Proposed by the Editor:

Find a finite number, x, so that the x-th power of x is a minimum.

No. 2—Proposed by the Editor:

A fox starts swimming at a given linear velocity from the center of a circular pool of given radius keeping his direction always toward a goose which is swimming with the same linear velocity around the edge of the pool. Will the fox overtake the goose? If so, when? Discuss or find the equation of his path.

SOME SIMPLE ASPECTS OF PRIMES

By S. T. SANDERS,
Louisiana State University

In a previous paper (See the December, 1930, issue of the Mathematics News Letter) we raised the question: If S is a fixed positive prime and p, q, variable distinct positive primes, are there primes S such that indefinitely many sets of values of p, q, exist, satisfying one of the relations $S=pq+p+q$, $S=pq-p+q$, $S=pq-p-q$?

We consider each of the cases separately

Case 1. $S=pq+p+q$, p and q positive.

We have: $pq=S-(p+q)$

As pq and $p+q$ are both positive $(p+q)$ cannot be greater than the fixed positive prime S, so that neither p nor q can be greater than S.

Hence, but a finite number of distinct primes at most can at most exist satisfying the relation

$$S=pq+p+q$$

Case 2. $S=pq-p+q$, p *and* q *positive.*

We have: $p(q-1)=S-q$.

Since S is a positive fixed prime and p, q are positive primes, inasmuch as 2 is the least positive value that S—q, or $p(q-1)$ can assume, it follows that an upper limiting value of q, under the conditions unposed is S—2. As p is a factor of S—q, p under the conditions must always be less than S—q. This proves that only a finite number of sets of distinct primes p, q can at most exist, satisfying the relation $pq-p+q=S$.

Case 3. $S=pq-p-q$, p, q positive.

We have: $p=\dfrac{S+q}{q-1}$, from which it is evident that 2 must be the minimum prime value of q, since if q were 1, p would not be finite.

To this minimum value of q we now show there corresponds a maximum value of p.

Let q' be any positive integer, prime or not prime, and let it be substituted for q in $\dfrac{S+q}{q-1}$. Denoting the corresponding value of the fractional form by p', we have

$$p'=\frac{S+q'}{q'-1} \qquad \text{(d)}$$

Again, in (d) put $(q'+1)$ for q'. We may then write,

$$p''=\frac{S+q'+1}{q'}$$

Then, we have $p''<p'$. For, if not, we should have

$$\frac{S+q'+1}{q'} \geqq \frac{S+q'}{q'-1}$$

or $(-S-1)\geqq 0$, which is false, since S is a positive integer.

This proves that as q is increased, p grows less, so that when q is a minimum prime, p is a maximum integer.

Putting 2 for q in $p=\dfrac{S+q}{q-1}$

we get $p=S+2$, the greatest value which p might possibly assume for a fixed prime S.

This proves that only a finite number of solutions can at most exist for *case* 3.

Calling the three different kinds of relation (a), (b), (c), we write:

(a) $p = \frac{S - q}{q + 1}$

(b) $p = \frac{S - q}{q - 1}$

(c) $p = \frac{S + q}{q - 1}$

In order to determine if a solution , or solutions, of any of the three relations exist, it is sufficient to assign a succession of prime values to q, beginning with 2, until a prime p is found, if such a prime exists.

The reader may check with the aid of one or the other of the three forms the following items which we shall call a

* *Supplement to Table* 1

$3 \cdot 31 + 3 + 31 = 127$
$2 \cdot 43 + 2 + 43 = 131$
$2 \cdot 139 - 2 - 139 = 137$
$2 \cdot 47 - 2 + 47 = 139$
$2 \cdot 151 - 2 - 151 = 149$
$3 \cdot 37 + 3 + 37 = 151$
$2 \cdot 53 - 2 + 53 = 157$
$3 \cdot 83 - 3 - 83 = 163$
$3 \cdot 41 + 3 + 41 = 167$
No solution exists for $S = 173$
$2 \cdot 59 + 2 + 59 = 179$
$2 \cdot 61 - 2 + 61 = 181$
$3 \cdot 47 + 3 + 47 = 191$
$2 \cdot 191 + 2 - 191 = 193$
$2 \cdot 199 - 2 - 199 = 197$
$3 \cdot 101 - 3 - 101 = 199$
$71 \cdot 2 + 71 - 2 = 211$
$3 \cdot 113 - 3 - 113 = 223$
$5 \cdot 37 + 5 + 37 = 227$
$2 \cdot 227 + 2 - 227 = 229$
$3 \cdot 59 - 3 + 59 = 233$

The Table is a continuation of Table 1, as published in the December, 1930, issue of the News Letter.

$2\cdot 79+2+79=239$
$3\cdot 61-3+61=241$
$2\cdot 83+2+83=251$
$3\cdot 127+3-127=257$
$5\cdot 43+5+43=263$
$2\cdot 89+2+89=269$
$2\cdot 269+2-269=271$
$5\cdot 47+47-5=277$
$3\cdot 71+71-3=281$
No solution for $S=283$
$2\cdot 97+2+97=293$
$2\cdot 103-2+103=307$
$2\cdot 103+2+103=311$
$3\cdot 79-3+79=313$
$3\cdot 157+3-157=317$
$3\cdot 167+3-167=337$
$2\cdot 347+2-347=349$
$3\cdot 89-3+89=353$
$2\cdot 349-349-2=347$
$11\cdot 29+11+29=359$
No solution exist for $S=367$
$7\cdot 61+7-61=373$
$2\cdot 127+127-2=379$
$2\cdot 127+127+2=383$
$3\cdot 193+3-193=389$
$5\cdot 67+67-5=397$
$3\cdot 101+101-3=401$
$2\cdot 137+137-2=409$
$2\cdot 139+139+2=419$
$2\cdot 419+2-419=421$
$5\cdot 71+71+5=431$
$2\cdot 431-431+2=433$
$3\cdot 109+109+3=439$
$5\cdot 73+5+73=443$
$2\cdot 149+2+149=449$
$13\cdot 37-37+13=457$
$3\cdot 229-229+3=461$
$7\cdot 79-7-79=467$
$5\cdot 79+5+79=479$

$$2\cdot 163+163-2=487$$
$$2\cdot 163+163+2=491$$
$$2\cdot 167+167-2=499$$
$$2\cdot 167+167+2=503$$

Thus with the exception of the primes 173, 283, 367, every prime in the interval from 3 to 503 is expressible in one or more of the forms $pq+p+q$, $pq-p+q$, $pq-p-q$, in which p, q are positive primes.

AN APPLICATION OF THE METHOD OF SUCCESSIVE SUBSTITUTIONS TO AN ANNUITY PROBLEM

By H. L. SMITH,
Louisiana State University

The problem of finding the rate of interest when the present value of an annuity of 1 per year for a given number of years is given receives very inadequate treatment in all of the textbooks on the theory of investment that the writer has seen. In this note a treatment based on the method of successive substitutions is given.

§1. **Analytic formulation of the problem**. Let p be the present value of an annuity of 1 for n years at rate r. Then

(1) $[1-(1+r)^{-n}]/r=p.$

Since the present value is at most n, our problem is to solve (1) for all positive values of r on the supposition that $0<p<n$.

Set

(2) $g_0(r)=[1-(1+r)^{-n}]/r-p,$

so that the equation (1) becomes

(3) $g_0(r)=0$

2. **The** *mth* **derivative of** g_0 **(***r***).** We may write g_0 (r) in the form given by the following equation

(4) $g_0(r)=(1+r)^{-1}+(1+r)^{-2}+\ldots+(1+r)^{-n}-p$

On differentiating this equation m times, we get

(5) $g_0^{(m)}(r)=(-1)^m[1.2\ldots m(1+r)^{-m-1}+2.3\ldots(m+1)(1+r)^{-m-2}+\ldots+n(n+1)\ldots(n+m-1)(1+r)^{-m-n}].$

From (5) we have

(6) $g_0^{(m)}(0)>g_0^{(m)}(r)>0 \qquad (m \text{ even}, r>0)$

(7) $g_0^{(m)}(0)<g_0^{(m)}(r)<0 \qquad (m \text{ odd}, r>0)$

We also find from (5)

(8) $\quad g_0'(0)=-(1+2+..+n)=-\frac{1}{2}n(n+1)$.

Finally from (4)

(9) $\quad g_0(0)=n-p>0$.

§3. Reformulation of the problem. Equation (3) is obviously equivalent to

(10) $\quad r=f(r)$

where

(11) $\quad f(r)=r-g(r)/M(r)$,

in which $g\ (r)$ is a constant times $g_0\ (r)$ and $M\ (r)$ is any function of r which does not vanish for $r>0$. Let us so choose $g\ (r)$ that $g'(0)=-1$. To do this it follows from (8) that we have only to define $g\ (r)$ by the following equation

(12) $\quad g(r)=[2g_0(r)]/[n(n+1)]$.

§4. The Maclaurin expansion of $g(x)$. By the aid of the binomial theorem, we find

(13) $\quad g(r)=a_0-a_1\ r+a_2\ r^2-a_3\ r^3+\ldots.$

where

$$\begin{aligned} &a_0=[2(n-p)]/[n(n+1)], \quad a_1=1 \\ (14)\quad &a_2=(n+2)/3, \quad a_3=[a_2(n+3)]/4, \\ &a_4=[a_3(n+4)]/5,\ldots\ldots. \\ &a_m=[a_{m-1}\cdot\ (n+m)]/(m+1),\ldots. \end{aligned}$$

This series certainly converges for $0<r<1$. Moreover, for such a value of r, by §2, the error made by taking as the value of $g(r)$ the first m terms on the right of (13) is $(-1)^m a_m er^m$, where $0<e<1$.

§5. A lemma on the method of successive substitutions. In a paper in this NEWS LETTER for January, 1931, the writer gave several sufficient conditions for the solution of equations of the form (11) by successive substitutions. One of these was stated as Theorem 5. We note here the following corollary to Theorem 5.

Theorem 5 *remains true if hypotheses* (H), (K) *are replaced respectively by* (H'), (K') *where* (H') *is as there given and* (K') *is as follows*:

$$\begin{aligned} (K')\quad &0<g'(x)/M(x)\leq 1 \qquad \text{if } 0\leq G\ g(x) \\ &0<g'(c)/M(x)\leq 1 \qquad \text{if } 0>G\ g(x) \end{aligned}$$

The truth of this corollary follows from Theorem 3 and equations (5), (8) of that paper.

§6. Application of the lemma. We have now the following *Theorem.* If $0<p<n$, the equation

$$[1-(1+r)^{-n}]/r=p$$

has exactly one positive solution for r, which may be obtained by applying the method of successive substitution to the equation

$$r=f(r)$$

where $f(r)=r-g(r)/M(r)$, $g(r)$ being defined by (12) and $M(r)$ being any continuous functions such that

$$M(r)\leq g'(r).$$

This follows at once from §2 and the lemma of §5.

§7. Actual procedure in the case in which $1-2^{-n}<p<n$. If $1-2^{-n}<p<n$, we have $g(1)<0$ and since by (9), §2, $g(0)>0$, it follows that $0<r<1$. Hence the Maclaurin series for $g(r)$ converges. The same is true for the Maclaurin series for $g'(r)$:

(15) $\quad g'(r)=-[1-b_1\, r+b_2\, r^2-\ldots]$

where

(16) $\quad b_1=2a_2,\ b_2=3a_3,\ b_3=4a_4+,\ldots\ldots$

Moreover the error made in taking the first m terms in the right of (15) for the value of $g'(r)$ is $-(-1)^m b_m e r^m$ where $0<e<1$.

Hence if we set

(17) $\quad M(x)=-[1-b_1\, r+b_2\, r^2-\ldots.+b_{2m}\, r^{2m}]$,

we will have, as required, $M(x)\leq g'(r)$. Hence the theorem of §6 holds with

(18) $\quad f(r)=r+\ [a_0-a_1\, r+a_2\, r^2-\ldots.]/[1-b_1\, r+b_2\, r^2-\ldots +b_{2m}\, r^{2m}]$,

where m is any positive integer. The larger m is chosen the more rapid will be the convergence of the successive approximations but the greater will be the amount of labor required to perform each substitution. A good value to choose for m in practise is $m=1$.

We are now in a position to state the precise steps to be taken in solving the equation (1).

(I) Determine as many of the numbers a_0, a_1,....as may be required.

(II) Find b_1, b_2.

(III) Compute a succession of numbers r_0, r_1, r_2,...by means of the formulas

$$r_0=0$$
$$r_1=f(r_0)=f(0)=a_0$$
$$r_2=f(r_1)$$
$$r_3=f(r_2),$$
$$\cdots\cdots\cdots\cdots$$

§8. **An example.** Let it be required to find the rate of interest per month for an annuity of 1 per week for 6 years whose present value is 200. This amounts to solving the equation

$$[1-(1+r)^{-312}]/r=200.$$

With the aid of a five-place table of logarithums we find the value of r to four significant figures (six decimal places) as follows.

Setting $n=312$, $p=200$, we find

$\log a_0=\ .36056\text{-}3$	$\log a_1=\ .00000$
$\log a_2=2.01981$	$\log a_3=3.91606$
$\log a_4=5.71678$	$\log a_5=7.43969$
$\log a_6=9.09702$	$\log a_7=10.69772$
$\log b_1=2.32084$	$\log b_2=4.39318$

Then

$$r_0=0 \qquad r_1=a_0=\ .0022938$$

A convenient form for finding r_2 is the following:

$$\begin{array}{ll} \log a_2 & =2.01981 \\ \log r_1^2 & =\ .72112\text{-}6 \\ \hline \log a_2\ r_1^2 & =\ .74093\text{-}4 \\ \hline \end{array} \qquad \begin{array}{ll} \log a_3 & =3.91606 \\ \log r_1^3 & =\ .08168\text{-}8 \\ \hline \log a_3\ r_1^3 & =\ .99774\text{-}5 \\ \hline \end{array}$$

$$\begin{array}{ll} \log a_4 & =5.71678 \\ \log r_1^4 & =\ .44224\text{-}11 \\ \hline \log a_4\ r_1^4 & =\ .15902\text{-}5 \end{array} \qquad \begin{array}{ll} \log a_5 & =7.43969 \\ \log r_1^5 & =\ .80280\text{-}14 \\ \hline \log a_5\ r_1^5 & =\ .24249\text{-}6 \end{array}$$

$$\begin{array}{rl} a_0 & =\ .002294 \\ a_2\ r_1^2 & =\ .000551 \\ a_4\ r_1^4 & =\ .000014 \\ \hline & \ .002859 \\ & \ .002395 \\ \hline & \ .000464 \end{array} \qquad \begin{array}{rl} r_1 & =\ .002294 \\ a_3\ r_1^3 & =\ .000099 \\ a_5\ r_1^5 & =\ .000002 \\ \hline & \ .002395 \end{array}$$

$\log b_2 \quad = 4.39318$ $\qquad\qquad$ $\log b_1 \quad = 2.32084$

$\log r_1^2 \quad = .72112\text{-}6$ $\qquad\qquad$ $\log r_1 \quad = .36056\text{-}3$

$\log b_2\ r_1^2 = .11430\text{-}1$ $\qquad\qquad$ $\log b_1\ r_1 = .68140\text{-}1$

$1 = 1.0000$

$b_2\ r_1^2 = .1301$

$-b_1\ r_1 = -.4802$

$.6499$

$$r_2 = .002294 + .000464/.6499$$
$$= .002294 + .000715 = .003009$$

On proceeding in this manner we get

$$r_3 = .003009 + .000045/.5940$$
$$= .003009 + .000078 = .003087$$
$$r_4 = .003087 + .000001 = .003088$$
$$r_5 = .003088 + .0000019/.5894$$
$$= .003088 + .0000032 = .0030912$$
$$r_6 = .0030912 + (-.0000003)/.59$$
$$= .0030912 - .0000005 = .0030907$$

The result $r = .003091$ is probably correct to the last figure.

THE ALGEBRAIC EQUATION AS A BALANCE

By H. E. BUCHANAN,
Tulane University

Definitions. Any combination of letters and symbols used to represent a number is called an **algebraic expression,** or simply an **expression.**

An **equation** is a statement of equality of two expressions. The expressions thus connected are called **members** of the equation. Equations, as defined above are of two kinds:

1. **Identical equations,** or **identities,** like $6X/3 = 2X$, which are true for every possible value of X.

2. **Conditional equations,** or **equations** which are true for

only one or at most for only a few values of the unknown letter and untrue for all others.

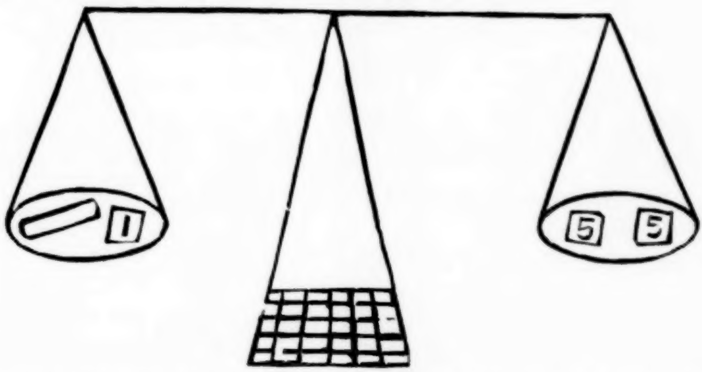
For example, $2X+1=5$ is true if $X=2$ and untrue for all other values of X. The values of the unknown which make a conditional equation true are said to **satisfy** the equation. They are called **roots** of the equation.

We use conditional equations and identities very often in algebra. Every time an expression is changed in form such as in factoring or addition of fractions the equal marks are used as in the identity. Thus,

$$2X(-X)=-2X^2$$

is an identity.

The Balance. Beginning students often experience some difficulty in handling the unknown quantity. They do not wish to work with it while its value is still unknown. They frequently ask: "What value has X?" "Is it 3?" "How can I use it without knowing its value?" To remove this fear of the unknown and at the same time to get a clear idea of the meaning of a conditional equation we consider the following problem.



A man wishes to weigh an iron window weight. He has a pair of balances as shown in the figure, two 5 lb. weights and one 1 lb. weight. This weight of the bar is unknown. The fact that it is unknown, however, does not make it strange or mysterious to us. We handle it just as we handle one of the known 5 lb. weights. We place the bar in one pan and find that it is lighter than two 5 lb. weights, but by trial we find that the bar and the 1 lb. weight are exactly balanced by the two 5 lb. weights. If

instead of the two 5 lb. weights we had ten 1 lb. weights we could evidently take one weight out of each pan and still keep a balance, i. e.,

The bar $=9$ lb.

In order to weigh the bar we must solve the equation

$W+1=10,$

where W stands for the weight of the bar. An equation viewed in this way is nothing but a balance. Anything can be done to an equation that will not destroy the balance. Evidently the same number can be added or subtracted from both members. Then the way to solve the equation

$$W+1=10$$

is to subtract $1=1$ from both members.

and obtain $W=9$.

Suppose now that a different iron bar is to be weighed and it is found that in order to get a balance two unknown weights must be placed in one pan and a 5 lb. and 1 lb. weight in the other. This situation is evidently represented by the equation

$2W=6.$

Clearly there would still be a balance if the weight in each pan were just half as great. That is we may divide both members of the equation by 2. This gives

$W=3$ lb.

Again suppose that two of the unknown weights and one 1 lb. weight are necessary to balance the two 5 lb. weights. This state of affairs is expressed by the equation

$2W+1=10.$

We may find W from this equation as follows:

$$2W+1=10,$$

Subtracting $1=1$ from both members

We have $2W=9$.

Dividing both members by 2 gives

$$W=4\tfrac{1}{2} \text{ lb.}$$

A General Principle. The above examples show very clearly that the following general rule holds true:

The form of an equation may be changed by adding the same number to both members, by subtracting the same number from both members or by multiplying or dividing both members by the same number.

(In dividing both members by the same number we must except the number 0.)

The above principle holds because the balance is not disturbed by any of the operations mentioned in it.

Conclusion. The author believes that the equation should be presented to elementary classes in algebra by the above method. If necessary many other examples can be made up. Transposition of numbers from one member of an equation to the other by changing the sign of the term transposed should be avoided till the student is well versed in handling the equation as a balance. For example:

Solve the equation $2X-\frac{1}{2}=X/4+3$.

Solution:	$2X-\frac{1}{2}$	$=X/4+3.$
Multiply by 4,	$8X-2$	$=X+12.$
Add 2	2	$=\quad 2$
	$8X$	$=X+14.$
Subtract X,	X	$=X$
	$7X$	$=14,$
Divide by 7	X	$=2,$

Check. Left member: $2(2)-\frac{1}{2}=4-\frac{1}{2}=3\frac{1}{2}$

Right member: $2/4+3=3\frac{1}{2}$.

Finally the reason why the sign of a number is changed, when it is transposed from one member to the other, should be brought out, if possible by the student himself, after many problems have been solved like the above.

CORRECTIONS.

The following corrections should be made in the January issue of the *News Letter*:

Page 23, line 2, change $f(x)$ to $g(x)$.

Page 23, line 5, change $g(x_0x_1)$ to $g(x_0x_1)/M(x)$.

Page 23, line 26, change $g_1(x)$ to $g^1(x)$.

Page 23, line 27, change $g(x)$ to $g^1(c)$.

Page 24, line 26, change $g(cr)\ g(cr)$ to $g(cx)\ g(cr)$.

Replace $\leqq$ by $\geqq$ in the following places: page 23, lines 26, 36; page 24, lines 11, 26.

On page 24, lines 11, 13, 28, 30, replace <1 by $\leqq 1$.

Baton Rouge, La., March 10th, 1931

MATHEMATICS NEWS LETTER
in account with
ORTLIEB PRESS

Mch. 24 Balance	$28.65	
Apr. 8 Inv. 1285	53.40	
May 5 " 1314	51.60	
June 30 " 1344	51.60	
July 30 " 1357	54.10	
Aug. 16 " 1367	3.00	
Oct. 10 " 1409	53.40	
Nov. 8 " 1436	53.40	
Dec. 13 " 1458	51.60	
Feb. 5 " 1523	53.40	
Mch. 7 " 1551	55.20	—509.35
Less Cash Payments		246.35
Balance		263.00

Paid in full by personal (ninety-day) note of S. T. Sanders.

Ortlieb Press,
R. A. ORTLIEB.

S. T. SANDERS,
in account with
MATHEMATICS NEWS LETTER
From March 24, 1930 to March 24, 1931

Received

Donations from individuals and institutions	$174.15
Subscriptions, new and renewed	$103.50
Proceeds from sale of back volumes of News Letter	$ 2.00
Total	$279.65

Expended

For stamped wrappers (used in mailing out News Letter)	$ 36.33
*For circulars and bill heads	5.10
*For envelopes and postage	10.96
For cuts	11.16
For partial cost of printing 9 issues of News Letter	246.35
Total	$309.90

*This item resulted from efforts to advertise the News Letter.

Amount of above named expenditures in excess of above named receipts, namely, $30.25 has been personally cared for by the Manager of the News Letter:

The total cost of printing 9 issues of the News Letter was	$509.35
On this debt was paid as shown above	246.35
Balance, due March 7, 1931,	$263.00

This balance was paid by the proceeds of a note given by the Manager.